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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2017
FIRST YEAR [BATCH 2017-20]

Date : 23/12/2017 **MATH FOR INDUSTRIAL CHEMISTRY [General]**

Time : 11 am – 2 pm **Paper : I** Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A

(Answer any five questions) [5×5]

1. Show that the product of all the four values of $\left(1+i\sqrt{3}\right)^{3/4}$ is 8. 5

2. If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0$, then find the equation whose roots are $\beta^2+\beta\gamma+\gamma^2,\ \gamma^2+\gamma\alpha+\alpha^2,\ \alpha^2+\alpha\beta+\beta^2$. 5

3. If the equation $ax^3+3bx^2+3cx+d=0$ has two equal roots then show that $(bc-ad)^2=4(b^2-ac)(c^2-bd)$ and each of the equal roots is $\dfrac{bc-ad}{2(ac-b^2)}$. 3+2

4. a) If $A=\begin{bmatrix}3&1&0\\2&1&4\\1&2&0\end{bmatrix}$ then find $A^T+\det A\cdot I$ where $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$. 2

   b) Solve using Cardan's method: $x^3+6x^2+11x+6=0$. 3

5. Consider the following system of equations: 5

$$\begin{aligned}2x+3y+az&=1\\x+\ y+5z&=6\\y+2z&=3\end{aligned}$$

   Find the values of a for which the above system has a unique solution.

6. Prove that every square matrix can be expressed as a sum of a symmetric matrix and a skew-symmetric matrix uniquely. 5

7. a) Define an orthogonal matrix and give an example. 2

   b) If A is an orthogonal matrix then what are the possible values of its determinant? Justify. 3

8. If $A=\begin{bmatrix}1&0&2\\0&-1&1\\0&1&0\end{bmatrix}$, then verify that A satisfies its own characteristic equation. Using this result, find A^{-1}. 3+2

Group – B

(Answer any five questions) [5×5]

9. Find the right and the left hand limits of the function: $f(x)=\begin{cases}\dfrac{|x-4|}{x-4}, & x\neq 4\\ 0\ , & x=4\end{cases}$.

Find also the value of $\lim_{x\to 4} f(x)$, if exists. 3+2

10. Let $f(x)=|x-1|+|x-2|^2+|x-3||x-4|$.

Find: a) $\underset{x\to 1}{Lt}\, f(x)$ b) $\underset{x\to 3\frac{1}{2}}{Lt}\, f(x)$ 2½X2

11. If $[x]$ denotes the largest integer less than equal to x, then discuss the continuity at $x = 3$ for the function $f(x)=x-[x],\ \forall x\geq 0$. 5

12. If a function f is continuous at an interior point c of an interval $[a, b]$ and $f(c)\neq 0$, then prove that there exists a $\delta>0$ such that $f(x)$ has the same sign as that of $f(c)$ for every $x\in(c-\delta, c+\delta)$. 5

13. State and prove Euler's theorem on homogeneous function for three variables. 5

14. A function f is defined as

$$f(x, y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x, y)\neq(0,0)\\ 0\ , & (x, y)=(0,0)\end{cases}.$$

Show that both the partial derivatives (f_x and f_y) exist at (0,0) but the function is not continuous there. 5

15. a) State Schwartz's theorem on commutative property of mixed derivatives. 2

b) For the function

$$f(x, y)=\begin{cases}\dfrac{x^2y^2}{x^2+y^2}, & (x, y)\neq(0,0)\\ 0\ , & (x, y)=(0,0)\end{cases}$$

show that $f_{xy}(0,0)=f_{yx}(0,0)$. 3

16. Prove that, by the transformations, $u=x-ct,\ v=x+ct$, the partial differential equation $\dfrac{\partial^2 z}{\partial t^2}=c^2\dfrac{\partial^2 z}{\partial x^2}$ reduces to $\dfrac{\partial^2 z}{\partial u\partial v}=0$. 5

Group – C

(Answer any five questions) [5×5]

17. Discuss the advantages and disadvantages of AM, GM and HM. 5

18. The frequency distribution of expenditure of 1000 families is given below:

Expenditure (Rs):	40–59	60–79	80–89	100–119	120–139
No. of families :	50	—	500	—	50

The mean and the median of the distribution are both 87.50. Determine the missing frequencies. 5

19. If a set of n values $x_1, x_2, \cdots x_n$ with frequencies $f_1, f_2, \cdots f_n$ respectively are given, show that their root-mean-square deviation is least when deviations are taken about their mean. 5

20. Discuss with example how skewness and Kurtosis determines the shape of a distribution. 5

21. A variable assumes the values 1, 2, …… n with corresponding frequencies 1, 2, ……n. Calculate the variance of the variable. 5

22. The first three moments of a distribution about the value 3 of a variable are 2, 10 and 30 respectively. Obtain the first three moments about zero. Find also the variance of the distribution. 5

23. Show that the value of correlation coefficient (r_{xy}) of two variables x and y always lies between –1 and 1. 5

24. Out of two lines of regression given by $x+2y-5=0$ and $2x+3y-8=0$, which one is the regression line of x on y? Use the equations to find the means of x and y and the correlation coefficient. If the variance of x is 12, calculate the variance of y. 5

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